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MULTI-SAMPLE ADAPTIVE TEST AND OTHER COMPETITORS IN LOCATION PROBLEM

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Abstract- Many adaptive tests have been developed in an effort to improve the performance of tests of significance. We will consider a test of significance to be "adaptive" if the test procedure is modified after the data have been collected and examined. For example, if we are using a certain kind of two-sample adaptive test we would collect the data and calculate selection statistics to determine which two-sample test procedure should be used. If the data appear to be normally distributed, then a Wilcoxon rank-sum test would be used; but if the data appear to contain outliers, then a median test would be used. Adaptive tests of significance have several advantages over traditional tests. They are usually more powerful than traditional tests when used with linear models having long-tailed or skewed distributions of errors. In addition, they are carefully constructed so that they maintain their level of significance. That is, a properly constructed adaptive test that is designed to maintain a significance level of α will have a probability of rejection of the null hypothesis at or near α when the null hypothesis is true. Hence, adaptive tests are recommended because their statistical properties are often superior to those of traditional tests. The adaptive tests have the following properties:

- The actual level of significance is maintained at or near the nominal significance level of α
- If the error distribution is long-tailed or skewed, the adaptive test is usually more powerful than the traditional test, sometimes much more powerful.
- If the error distribution is normal, there is little power loss compared to the traditional tests.
- Adaptive tests are practical.

Theoretical statisticians have all too often accepted a model and considered many statistical inferences using that model without checking its validity. That is, we have accepted a dogma of normal distribution and routinely performed the appropriate statistical inference. Possibly a better way to proceed would be to assume that an appropriate model is to found among a number of models, say $\omega_1, \omega_2, \ldots \omega_k$, which are suitably placed throughout the spectrum of possible models. Then use the data to select the model which seems most appropriate and with his model and the same data, make the desired inference. So, the aim of my paper is to make a comparison of power, among different adaptive tests and traditional tests, multi sample cases considering equal and unequal sample sizes.

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1.1 INTRODUCTION:

If the parametric assumptions are fulfilled, classical F test is the appropriate test for the multi sample location problem. On the other hand, if assumptions not satisfies ,F-test is not a suitable tests. So we have to search for some other tests. Test based on ranks or scores are found to be more powerful and robust in many situations. It is seen that many of the practicing statisticians have no idea regarding the data type. In this situations, the adaptive tests based on Hogg's concept , may help the statistician to identify data type with respect to some measures like skewness and tail weight and then to select an appropriate rank test or test based on scores for classified type of distribution. In this chapter, we have first discussed the adaptive test procedure that are used to select an appropriate test and then compare an adaptive test and some of the tests procedure s with the help of Monte Carlo simulation technique. Both empirical level and power of these tests are calculated and comparison are made with F-test and different other adaptive tests. We have observed that adaptive tests behave well in broad class of distributions.

1.2: Review of Adaptive Tests

An adaptive distribution-free test on linear statistics was suggested by Hajek(1962,1970). In his original scheme, Hajek estimated the optimal score function for the two-sample location problem in a consistent fashion. However, the slow convergence of his estimator rendered the procedure impractical. Gastwirth's (1965) simple modified tests paved the way for adaptive rank tests. Hogg (1967) proposed an adaptive procedure that used the sample kurtosis to select one of four estimators of the mean of a symmetric distribution. In that research, four symmetric distributions were considered having various amounts of kurtosis. The idea was to use the selection statistic to select an estimator that would have low variance for samples from that distribution. One difficulty with this approach is that the sample kurtosis is highly variable, so it may sometimes fail to select the correct estimator for that symmetric distribution. In spite of this problem, the robust adaptive estimator had excellent performance with n = 25 observations that were generated from the four distributions that were used in that study. In arguing for greater use of these robust methods, Hogg (1967) stated "In this age of excellent computing devices, the statistician should take a broader view and not select a narrow model prior to observing the sample items". Simpler schemes were subsequently developed by Randles and Hogg(1973) for both the one and two-sample location problems. Berens(1974) suggested an alternative estimation scheme to replace the one used by Hajek. Policello and Hettmansperger (1976) proposed an adaptive rank test for the one-sample location problem that is not distribution-free but maintains its α - levels reasonably well. Jones(1979) considered a different adaptive rank test that is distribution-free for that same one-sample problem. Adaptive rules proposed for other statistical problems include those due to Hogg and Randles(1975) and Hogg(1976). The first two-sample adaptive test that was practical and relatively powerful was proposed by Hogg, Fisher, and Randles (1975). Prior to 1975, the adaptive tests were interesting but not too practical. For example, the test proposed by Hajek (1962) was designed to improve the power by finding scores that would produce a locally most powerful rank test. The test required an estimate of the density function (f) and the first derivative of the function (f '). The problem with this approach is that f and $f^{/}$ are difficult to estimate unless the samples are very large. Hence, these adaptive tests are not practical and do not appear to be used. A general discussion and bibliography of adaptive inference was given by Hogg(1974).

Last 20 years after the paper by Hogg, Fisher, and Randles (1975), several researchers used the same selection statistics to construct tests for a variety of situations. Over the following years this estimator has been modified and the more recent version of this adaptive estimator, as described by Hogg and Lenth (1984), has excellent properties. Ruberg (1986) proposed a continuously adaptive two-sample test and O'Gorman (1997) proposed a continuously adaptive test for the one-way layout. Using a different approach, Hall and Padmanabhan (1997) proposed several adaptive tests for the two-sample scale problem. They used a bootstrap testing approach with adaptively trimmed sample variances. We have noted that in the last 40 years there has also been work in the area of adaptive estimation. Yuh and Hogg (1988) proposed two adaptive regression estimators that rely on selection statistics to choose one of several robust regression estimators. Further work in the area of adaptive estimation was published by Hill , Padmanabhan and Puri (1991), who described the performance of some adaptive estimators when they were used with real data.

Buning (1996) proposed an adaptive test of equality of medians using data from a one-way layout. This test was based on an extension of Hogg's method of using selection statistics to select a set of rank scores. Two years later, Buning and Kossler (1998) proposed an adaptive test for umbrella alternatives and, in the following year, Buning (1999) proposed a test for ordered alternatives.

Further extensions of the adaptive approach were made by Buning and Thadewald (2000), who proposed a location-scale test and by Buning (2002), who proposed a test that could be used to test the null hypothesis that the distributions are equal against the general alternative that the distributions are not equal.

The tests proposed by Hogg and by Buning used selection statistics to determine
the set of rank scores for the two-sample test. One small problem with this approach is that, if the selection statistics fall near the edge of a region corresponding to a set of rank scores, any small change in the data could change the selection statistics
slightly, and this could result in the selection of an entirely different set of rank scores. This is undesirable because a small change in a single data value could result in a large change in the *p*-value.

While most of the adaptive testing literature prior to 2000 focused on two-sample tests, some recent research has been published on one-sample adaptive tests. Freidlin, Miao, and Gastwirth (2003) proposed an interesting and effective adaptive test for paired data. These authors use the ***p***- value from a test of normality, rather than a measure of skewness or tail-weight, as the selection statistic. They showed that their test is reasonably effective for moderate sample sizes .Most recently, Miao and Gastwirth (2009) proposed a test that uses the same score functions that were used by Freidlin, Miao, and Gastwirth (2003), but the test uses a measure of tail-heaviness as the selection statistic.

A different approach to robustifying and improving two-sample tests was taken by Neuhauser, Buning, and Hothorn (2004). To construct their test, they used four sets of rank scores to produce four standardized linear rank statistics. Next, they
computed the maximum of those four statistics as the overall test statistic, which is then used with a permutation method to compute the *p*-value. This test maintains its level of significance and has higher power than many of the traditional parametric
and nonparametric tests. In addition, it has the advantage of not using any selection statistic. While it is not always classified as an adaptive test, it does achieve the same objective as the adaptive test.

Hao and Houser(2012) proposed some adaptive procedure for WMW test: seven Decades of Advances. Here it is discussed that the WMW test has dominated non parametric analysis is behavioural sciences for the past seven decades. Its widespread use makes the fact that there exist simple "adaptive" procedures which use data dependent statistical decision rules to select an optimal non parametric test. In this dissertation, key adaptive approaches for testing differences in locations in two sample and multi-samples environments are considered.

1.4 Selection Statistics:

Here we will use a selection statistics S =(Q_1,Q_2), where Q_1 and Q_2 are Hoggs measure of skewness and tailweight defined by –

$$Q_1 = \frac{\bar{U}_{5\%}-\bar{M}_{50\%}}{\bar{M}_{50\%}-\bar{L}_{5\%}} \text{ and } Q_2 = \frac{\bar{U}_{5\%}-\bar{L}_{5\%}}{\bar{U}_{50\%}-\bar{L}_{50\%}}$$

Where , $\bar{U}_{5\%}$, $\bar{M}_{50\%}$ and $\bar{L}_{5\%}$ are the averages of the upper 5% , middle 50% and lower 5% of the order statistics of the combined sample. $\bar{U}_{50\%}$ and $\bar{L}_{50\%}$ are the averages of the upper 50% and lower 50% of the order statistics of the combined sample

Table 1.1: Theoretical values of Q_1 and Q_2 for some selected distributions-

Distributions	Q_1	Q_2
Uniform	1	1.9
Normal	1	2.585
Logistic	1	3.204
Double exponential	1	3.302
Exponential	4.569	2.864

Now let us define four categories of S-

D_1 = {S/0≤Q_1≤2, 1≤Q_2 ≤2}

D_2 = {S/0≤Q_1 ≤2; 2≤Q_2 ≤3}

D_3 = {S/ Q_1≥0; Q_2 >3}

D_4 = {S/ Q_1>2; 1≤Q_2 ≤3}

This means that the distribution is short or medium tails if S falls in the

Category D_1 or D_2 respectively; long tail if S falls in the category D_3 and right skewed tail if it falls in the category D_4

Buning (1996) proposed the following adaptive test A :

$$A = \begin{cases} G & if \quad S \in D_1 \\ KW & if \quad S \in D_2 \\ LT & if \quad S \in D_3 \\ HFR & if \quad S \in D_4 \end{cases}$$

1.5 Test Procedures :

Let $X_{i1}, X_{i2}, \dots, X_{in_i}$, i =1,2,…,c be independent random variables with absolutely continuous distribution function F(x-θ_i).

Here the null hypothesis H_0:$\theta_1 = \theta_2 = \cdots = \theta_c$

Against the alternative hypothesis H_1:$\theta_r \neq \theta_s$ for at least one pair (r,s), r≠s.

1.5.1 F-Test:

For normally distributed random samples with equal variance, in testing equality of means the likelihood ratio F test is the best one. The test statistics defined as

$$F = \frac{(N-c)\sum_{i=1}^{C} n_i(\bar{X_i}-\bar{X})^2}{(C-1)\sum_{i=1}^{C}\sum_{j=1}^{n_i}(X_{ij}-\bar{X_i})^2}$$

where $N=\sum_{i=1}^{c} n_i$, $\bar{X}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} X_{ij}$, $\bar{X} = \frac{1}{N}\sum_{i=1}^{c} n_i \bar{X}_i$

Under H_0 the test statistics follows F distribution with c-1 and N-c degrees of freedom.

1.5.2 Kruskal-Wallis(KW) test:

Let R_{ij} be the rank of the observation x_{ij} in the pooled sample. The Kruskal-Wallis test for two-sided alternative which based on the statistic

$$\textbf{KW} = \frac{12}{N(N+1)}\sum_{i=1}^{k}\frac{1}{n_i}\left[R_i - \frac{n_i(N+1)}{2}\right]^2$$

$$KW = \frac{12}{N(N+!)}\sum_{i=1}^{c}\frac{1}{n_i}[R_i - \frac{n_i(N+1)}{2}]^2$$

$$= \frac{12}{N(N+1)}\sum_{i=1}^{c}\frac{R_i^2}{n_i} - 3(N+1)$$

where $R_i = \sum_{j=1}^{n_i} R_{ij}$ and $N = \sum_{i=1}^{c} n_i$.

For sample size n_i and sample number k large , H_o is rejected if KW> $\chi^2_{\alpha,(k-1)}$. When k is small and n_i are small then exact distribution table of KW can be used.

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Now let us define linear rank statistics. Let us consider a the combined ordered sample $X_{(1)}, X_{(2),...}X_{(N)}$ of $X_{11}, ...X_{1n_{1},...}X_{c1}, ...X_{cn_c}$ and indicator variables V_{ik} given by

$$V_{ik} = \begin{cases} 1 \; if \; X_{(k)} \; belong \; to \; the \; ith \; sample \; , i = 1, \dots, c, k = 1, \dots, N \\ 0 \quad otherwise \end{cases}$$

where $N=\sum_{i=1}^{c} n_i$,

Let a(k) , k =1,2,…,N be real valued score with mean $\bar{a} = \frac{1}{N}\sum_{k=1}^{N} a(k)$. now we define for each sample a statistics A_i in the following way:

$$A_i = \frac{1}{n_i}\sum_{k=1}^{N} a(k)V_{ik}, \; 1 \le i \le c$$

Then the linear rank statistics L_N is given by

$$L_N = \frac{(N-1)\sum_{i=1}^{c} n_i (A_i - \bar{a})^2}{\sum_{k=1}^{N} (a(k) - \bar{a})^2}$$

Under H_0 ,L_N is distribution free and follows asymptotically chi- square distribution with c-1 degrees of freedom .

Some of the scores to obtain more powerful test for types of distribution according to Buning(1991,1994) are as follows:

1.5.3 Gastwrith Test g (short tails):

$$a_G(k) = \begin{cases} k - \frac{(N+1)}{4} & if \; k \le \frac{(N+1)}{4} \\ 0 \quad if \; \frac{(N+1)}{4} \le k \le \frac{3(N+1)}{4} \\ k - \frac{3(N+1)}{4} & if \; k \ge \frac{3(N+1)}{4} \end{cases}$$

1.5.4 Kruskal Wallis test KW (medium tails):

If $a_{KW}(k) = k$, test transform to above KW test

1.5.5 Test LT (long tails):

$$a_{LT} = \begin{cases} -(\left[\frac{N}{4}\right] + 1 & if \quad k < \left[\frac{N}{4}\right] + 1 \\ k - \frac{(N+1)}{2} & if \quad \left[\frac{N}{4}\right] + 1 \le k \le [\frac{3(N+1)}{4}] \\ \left[\frac{N}{4}\right] + 1 & if \quad k > [\frac{3(N+1)}{4}] \end{cases}$$

1.5.6 Hogg Fisher Randles test HFR(right skewed):

$$a_{HFR} = \begin{cases} k - \frac{(N+1)}{2} & if \quad k \le \frac{(N+1)}{2} \\ 0 & if \quad k > \frac{(N+1)}{2} \end{cases}$$

For left-skewed distributions we change the terms k-(N+1)/2 and 0 in the definition of the scores above.

1.6 Monte Carlo simulation:

We investigate the power of the tests via Monte Carlo simulation. For this purpose we have repeated 10000 times. The criteria of the test comparisons are the level α and the power β of the tests. The concept of α robustness can be defined as follows. For a nominal level α and underlying distribution function F, the critical region C_α of a statistic T may be uniquely determined by $P_{H0}(T \in C_\alpha ! F) = \alpha$. We now assume a distribution function for the data and determine the actual level α^* of the test, i.e. $\alpha^* = P_{H0}(T \in C_\alpha ! G)$, T is then called '$\alpha - robust$' if $!\alpha - \alpha^*!$ is small. In case of $\alpha^* \le \alpha$, we call the test conservative; otherwise , it is anticonservative.

The selected distributions for the robustness and power study are Normal, Logistic, Cauchy, Lognormal, Double Exponential, Exponential and Uniform. We consider cases of three samples and Four samples with sample size combinations (10,10,10),(10,15,20), (10,10,10,10) and (10,15,20,25). Various combinations of location parameters are considered which are shown in respective Tables. For generating the samples from the normal distribution formula given by Hammersley and Hanscomb(1964) is used and for other distributions we have used method inverse integration. Necessary modification are made in generated sample to represent the location shift.

Table 1.2 Empirical Level and power of tests under Normal distribution:

Sample sizes n_i	Location parameter μ_i	F 5%	F 1%	H 5%	H 1%	G 5%	G 1%	LT 5%	LT 1%	HFR 5%	HFR 1%
10 10 10	0 0 0	.0509	.0101	.0446	.0072	.0417	.0043	0482	.0067	.0451	.0059
	0 -.2 .2	.1216	.0678	.0969	.0230	.0756	.0142	.0812	.0210	.0714	.0110
	0 -.5 .5	.4636	.2247	.4254	.1681	.3607	.1032	.3908	.1598	.3436	.1149
	0 -1 1	.9754	.8926	.9643	.8290	.9059	.624	.9379	.7768	.8902	.6476
	0 -1.5 1.5	1.0	.9995	.9999	.9998	.9981	.9551	1.000	.9943	.9973	.9698
10 15 20	0 0 0	.0458	.0096	.0452	.0079	.0430	.0070	.0467	.0075	.0451	.0069
	0 -.2 .2	.1923	.0746	.1414	.0388	.1056	.0212	.1246	.0244	.0912	.0194
	0 -.5 .5	.7213	.4651	.6846	.4042	.6133	.307	.6345	.3674	.6007	.3361
	0 -1 1	.9997	.9952	.9997	.9913	.9972	.9601	.998	.9829	.996	.9719
	0 -1.5 1.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
10 10 10 10	0 0 0 0	.0484	.0085	.0424	.0055	.0414	.0053	.0453	.0069	.0421	.005
	-.1 .1 -.2 .2	.1042	.0420	.0694	.0120	.0476	.0094	.0544	.0102	.0420	.0078
	-.5 .5 -1 1	.9850	.9329	.9790	.8922	.9416	.7229	.9645	.8528	.9294	.7414
	-1 1 -1.5 1.5	1.000	1.000	1.000	1.000	1.000	.9908	.9998	.9959	1.000	.9973
	-1.5 1.5 -2 2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
10 15 20 25	0 0 0 0	.0485	.0101	.0452	.0091	.0492	.0063	.047	.0082	.0463	.0083
	-.1 .1 -.2 .2	.2254	.0842	.1610	.0480	.1256	.0410	.1422	.0376	.0968	.0216
	-.5 .5 -1 1	1.000	.9997	1.000	1.000	.9999	.9974	1.000	.9987	.9999	.998
	-1 1 -1.5 1.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	-1.5 1.5 -2 2	1.000	1.000.	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Fig-1.1 Empirical power of tests under Normal distribution for $n_1=n_2=n_3=10$ at 5% level:

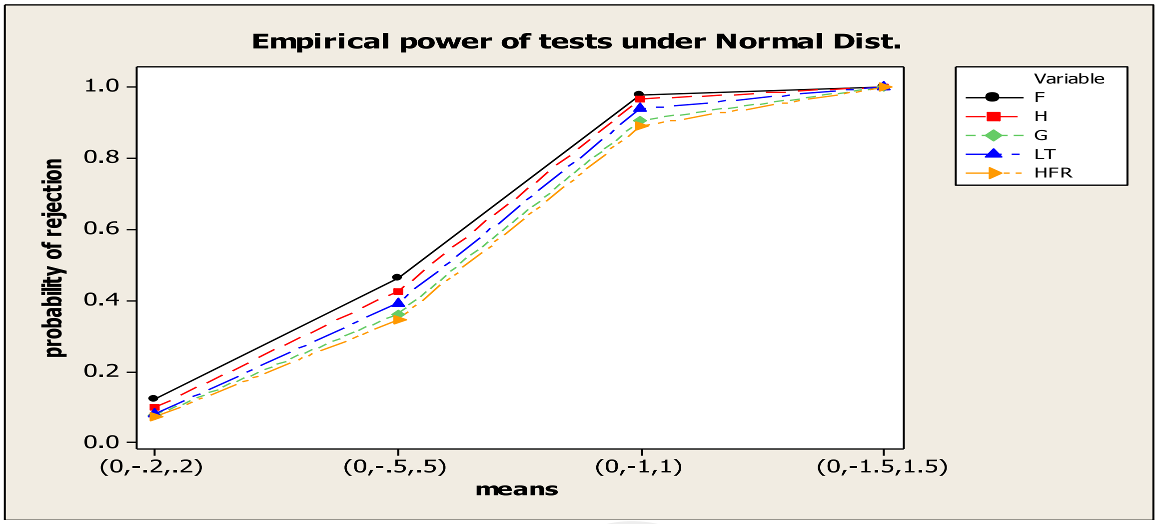

Fig-1.2 Empirical power of tests under Normal distribution for $n_1=10$, $n_2=15$, $n_3=20$, $n_4=25$ at 5% level:

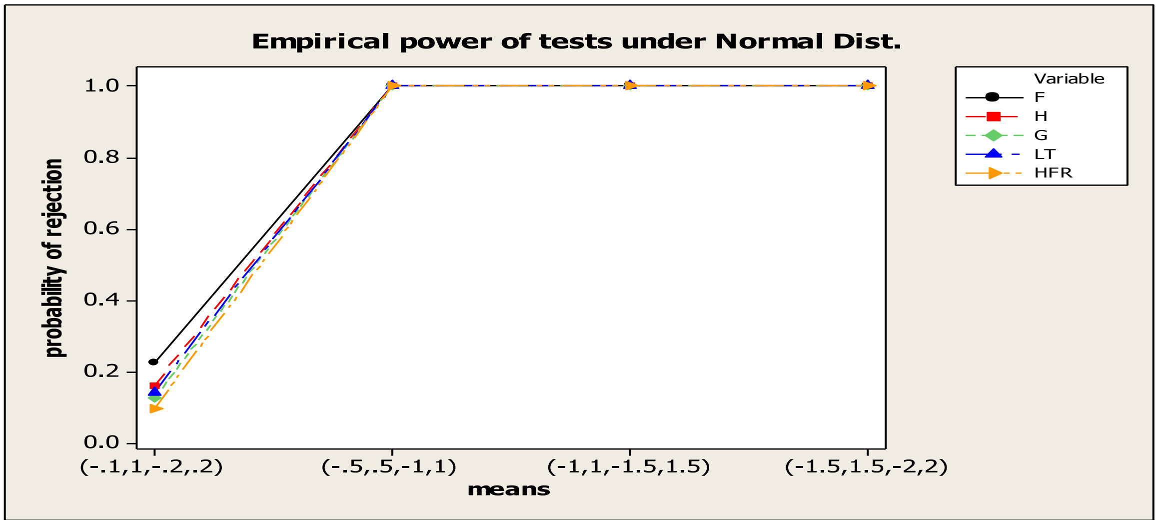

Table 1.3 Empirical Level and power of tests under Cauchy distribution:

Sample sizes n_i	Location parameter μ_i	F 5%	F 1%	H 5%	H 1%	G 5%	G 1%	LT 5%	LT 1%	HFR 5%	HFR 1%
10 10 10	0 0 0	.0196	.0025	.0454	.0068	.0390	.0037	.0471	.0069	.0342	.0035
	0 -.2 .2	.0248	.0028	.0642	.0120	.0476	.0040	.0656	.0154	.0598	.0084
	0 -.5 .5	.0420	.0072	.1536	.0430	.0683	.0089	.1698	.0478	.0999	.0057
	0 -1 1	.1136	.0398	.4920	.1878	.1582	.0359	.4966	.2350	.2331	.0234
	0 -1.5 1.5	.3014	.1780	.6660	.4058	.2808	.0884	.7629	.4969	.3499	.0022
10 15 20	0 0 0	.0254	.0014	.0456	.0066	.0436	.0055	.0456	.0075	.0423	.0408
	0 -.2 .2	.0286	.0128	.0792	.0154	.0518	.0084	.0832	.0156	.0764	.0120
	0 -.5 .5	.0488	.0092	.2492	.0882	.0997	.0203	.3034	.1136	1438	.0461
	0 -1 1	.1318	.0462	.6656	.3996	.2509	.0838	.7689	.5246	.3755	.1154
	0 -1.5 1.5	.2470	.1250	.8894	.7218	.4268	.1945	.9510	.8338	.5250	.3026
10 10 10 10	0 0 0 0	.0148	.0014	.0444	.0050	.0396	.0045	.0446	.0074	.0353	.0027
	-.1 .1 -.2 .2	.0170	.0018	.0618	.0096	.0439	.0062	.0602	.0114	.0510	.0076
	-.5 .5 -1 1	.0840	.0232	.4532	.2096	.1507	.0350	.5455	.2758	.2646	.1066
	-1 1 -1.5 1.5	.1922	.0875	.7754	.5158	.2814	.0932	.8699	.6618	.3549	.2256
	-1.5 1.5 -2 2	.3012	.1814	.9142	.7398	.4051	.1620	.9663	.8733	.5805	.3877
10 15 20 25	0 0 0 0	.0266	.0044	.0444	.0068	.0465	.0067	.0424	.0065	.0408	.0257
	-.1 .1 -.2 .2	.0302	.0048	.0780	.0150	.0568	.0102	.0880	.0170	.0816	.0184
	-.5 .5 -1 1	.1042	.0324	.7386	.4950	.2854	.1066	.8841	.7042	.4807	.2582
	-1 1 -1.5 1.5	.2188	.1116	.9588	.8618	.5201	.2726	.9941	.9672	.6443	.4330
	-1.5 1.5 -2 2	.3324	.2108	.9944	.9680	.7036	.4453	.9996	.9970	.8434	.6708

Fig 1.3 Empirical power of tests under Cauchy distribution for

$n_1=n_2=n_3=10$ at 5% level:

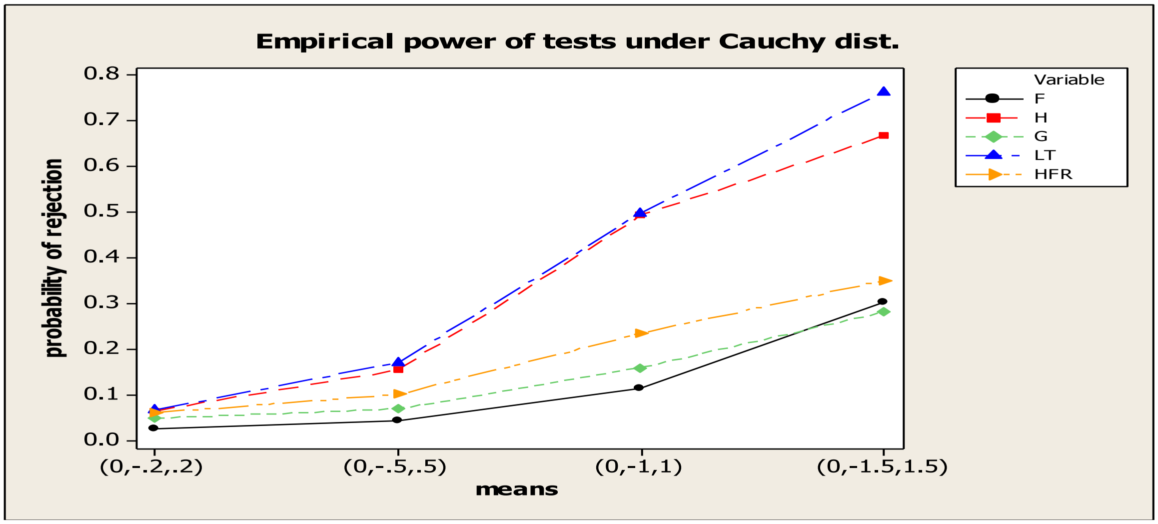

Fig.1.4Empirical power of tests under Cauchy distribution for $n_1=10$, $n_2=15$, $n_3=20$, $n_4=25$ at 5% level:

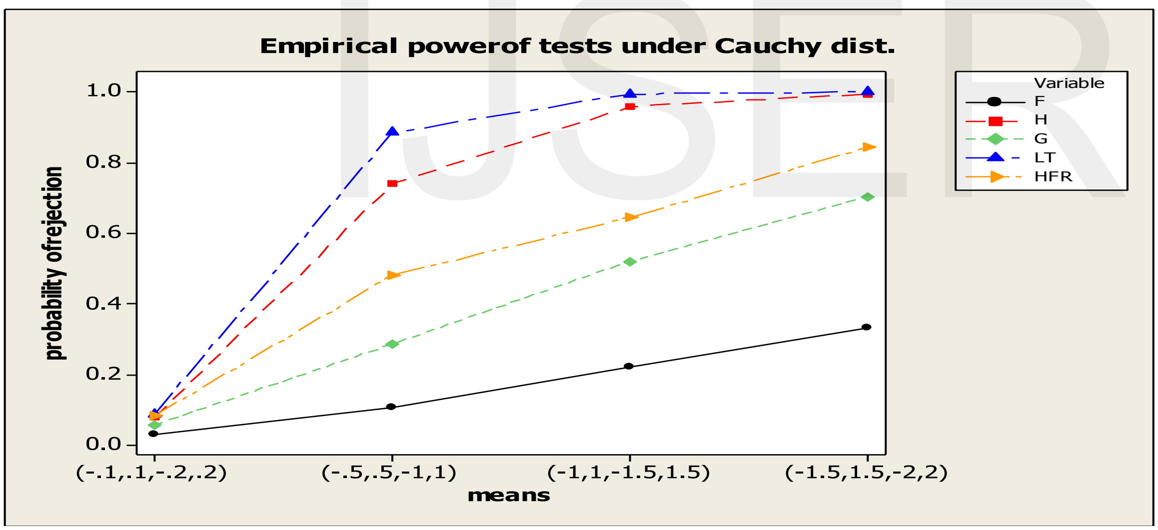

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Table 1.4 Empirical Level and power of tests under Logistic distribution:

Sample sizes n_i	Location parameter μ_i	F		H		G		LT		HFR	
		5%	1%	5%	1%	5%	1%	5%	1%	5%	1%
10 10 10	0 0 0	.0480	.0102	.0454	.0068	0439	.0039	. 0455	.0076	.0430	.0076
	0 -.2 .2	.0698	.0152	.0480	.0132	.0588	.0070	.0624	.0138	.0586	.0100
	0 -.5 .5	.1668	.0550	.1690	.0462	.1236	.0208	.1593	.0452	.1363	.0316
	0 -1 1	. 5496	.3050	.5528	.2636	.4036	.1316	.5299	.2516	.4448	.1737
	0 -1.5 1.5	.8828	.6990	.8826	.6502	.7373	.3895	.8694	.6373	.7805	.4914
10 15 20	0 0 0	.0452	.0089	.0456	.0066	.0494	.0059	.0462	.0072	.0477	.0060
	0 -.2 .2	.0860	.0202	.0820	.0158	.0744	.0116	.0816	.0156	.0764	.0176
	0 -.5 .5	.2816	.1168	.2848	.1054	.2138	.0620	.2654	.0985	.2380	.0864
	0 -1 1	.8104	.5970	.8228	.5844	.6723	.3688	.7955	.5571	.7370	.4821
	0 -1.5 1.5	.9892	.9522	.9908	.9504	.9487	.7888	.9870	.9367	.9729	.8889
10 10 10 10	0 0 0 0	.0488	.0088	.0444	.0050	.0407	.0040	.0454	.0068	.0436	.0070
	-.1 .1 -.2 .2	.0640	.0120	.0638	.0092	.0546	.0066	.0608	.0102	.0538	.0094
	-.5 .5 -1 1	.5832	.3318	.5914	.3088	.4315	.1512	.5758	.2974	.4767	.1978
	-1 1 -1.5 1.5	.9488	.8398	.9576	.8312	.8352	.5132	.9496	.8228	.8873	.6601
	-1.5 1.5 -2 2	.9994	.9922	.9996	.9904	.9744	.8150	.9985	.9887	.9903	.9393
10 15 20 25	0 0 0 0	.0544	.0098	.0484	.0078	.0476	.0080	.0447	.0080	.0434	.0071
	-.1 .1 -.2 .2	.0920	.0234	.0842	.0206	.0762	.0170	.0832	.0212	.0800	.0184
	-.5 .5 -1 1	.9020	.7518	.9200	.7638	.7966	.5482	.9055	.7401	.8613	.6531
	-1 1 -1.5 1.5	.9998	.9950	1.0	.9968	.9929	.9479	.9996	.9956	.9979	.9866
	-1.5 1.5 -2 2	1.000	1.000.	1.000	1.000	1.000	.9985	1.000	1.000	1.000	.9996.

Fig. 1.5 Empirical power of tests under Logistic distribution for $n_1=n_2=n_3=10$ at 5% level:

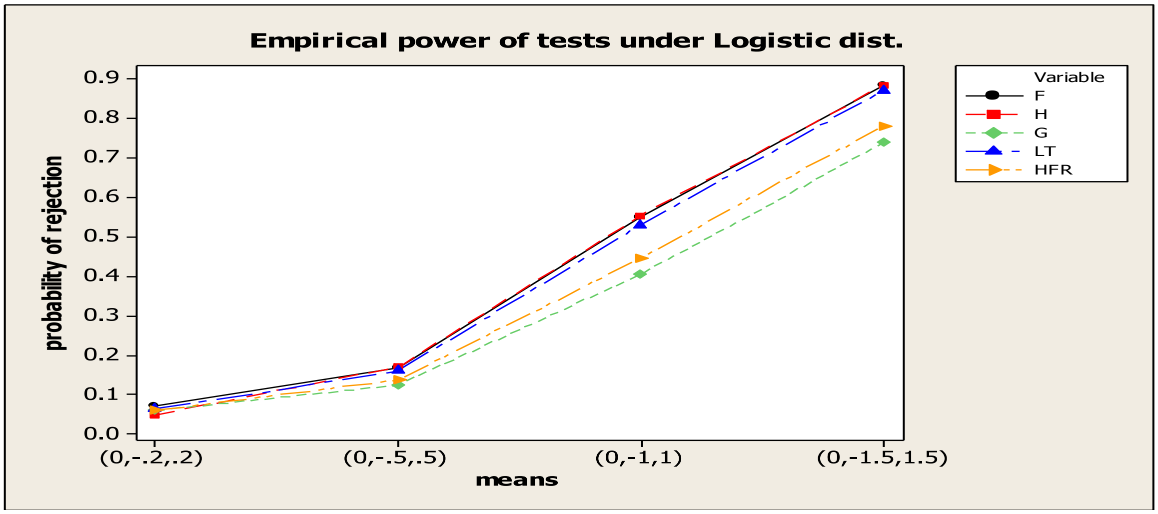

Fig. 1.6 Empirical power of tests under Logistic distribution for $n_1=10$, $n_2=15$, $n_3=20$, $n_4=25$ at 5% level:

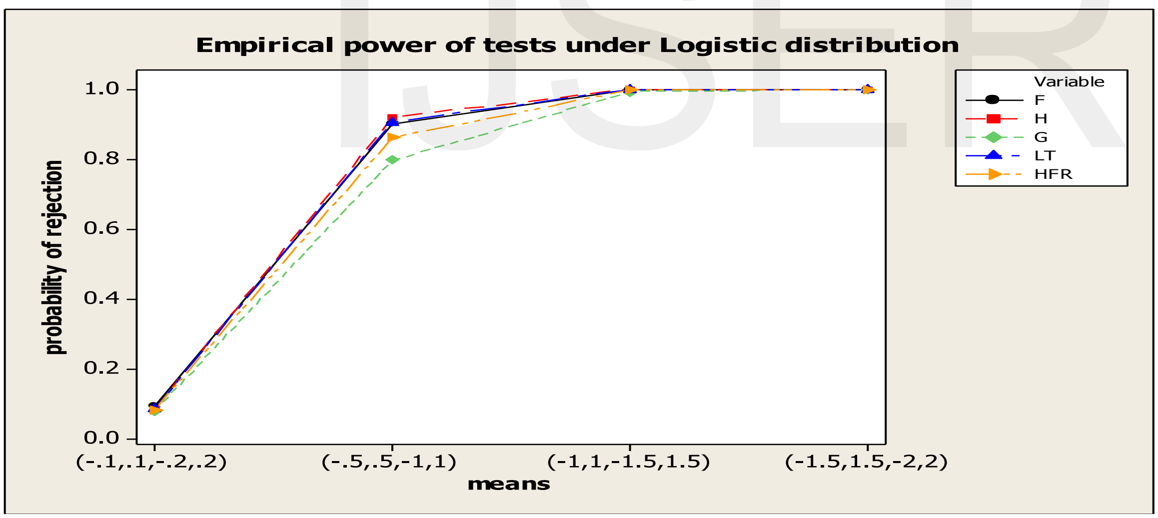

Table 1.5 Empirical Level and power of tests under Lognormal distribution:

Sample sizes n_i	Location parameter μ_i	F 5%	F 1%	H 5%	H 1%	G 5%	G 1%	LT 5%	LT 1%	HFR 5%	HFR 1%
10 10 10	0 0 0	.0353	.0057	.0434	.0068	.0417	.0043	.0482	.0067	.0451	.0059
	0 -.2 .2	.0726	.0144	.0966	.0232	.1554	.0268	.1174	.0262	.1898	.0482
	0 -.5 .5	.2759	.0902	.4258	.1660	.4094	.1543	.4501	.2046	.6970	.3646
	0 -1 1	.7665	.4559	9626	.8246	.7305	.4156	.8928	.6954	.9830	.8633
	0 -1.5 1.5	.9212	.7287	.9998	.9982	.8821	.6101	.9879	.9277	.9995	.9793
10 15 20	0 0 0	.0389	.0050	.0460	.0094	.0430	.0070	.0467	.0075	.0451	.0069
	0 -.2 .2	.0862	.0156	.1414	.0388	.2610	.0846	.1602	.0506	.3450	.1348
	0 -.5 .5	.4228	.1469	.6750	.3972	.6941	.4009	.6523	.4148	.9211	.7703
	0 -1 1	.9095	.6806	.9968	.9900	.9307	.7611	.9777	.9238	.9993	.9947
	0 -1.5 1.5	.9656	.8568	1.000	1.000	.9831	.9152	.9988	.9932	1.000	.9998
10 10 10 10	0 0 0 0	.0338	.0057	.0426	.0056	.0414	.0053	.0453	.0069	.0421	.0050
	-.1 .1 -.2 .2	.4974	.2476	.0852	.0166	.7392	.4468	.9350	.7966	9356	.5870
	-.5 .5 -1 1	.7841	.4927	.9790	.8922	.7336	.4355	.9308	.7866	.9925	.9488
	-1 1 -1.5 1.5	.9302	.7623	1.000	1.000	.8662	.6042	.9959	.9788	1.000	.9985
	-1.5 1.5 -2 2	.9486	.8157	1.000	1.000	.9295	.7273	.9996	.9985	1.000	.9999
10 15 20 25	0 0 0 0	.0420	.0082	.0480	.0087	.0492	.0063	.0470	.0082	.0463	.0083
	-.1 .1 -.2 .2	.7376	.5124	.1610	.0480	.9876	.9320	.9962	.9854	.9972	.9688
	-.5 .5 -1 1	.9595	.8302	1.000	.9996	.9871	.9301	.9960	.9840	1.000	1.000
	-1 1 -1.5 1.5	.9848	.9280	1.000	1.000	.9981	.9809	1.000	.9996	1.000	1.000
	-1.5 1.5 -2 2	.9883	.9435	1.000	1.000	.9993	.9944	1.000	1.000	1.000	1.000

Fig. 1.7 Empirical power of tests under Lognormal distribution for $n_1=n_2=n_3=10$ at 5% level:

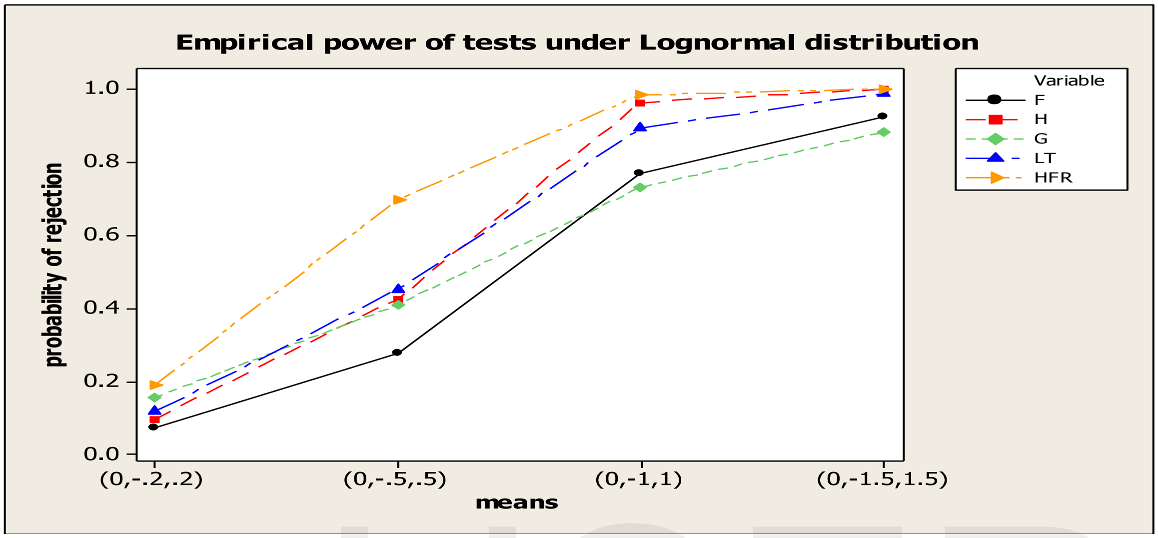

Fig. 1.8 Empirical power of tests under Lognormal test for $n_1=10$, $n_2=15$, $n_3=20$, $n_4=25$ at 5% level:

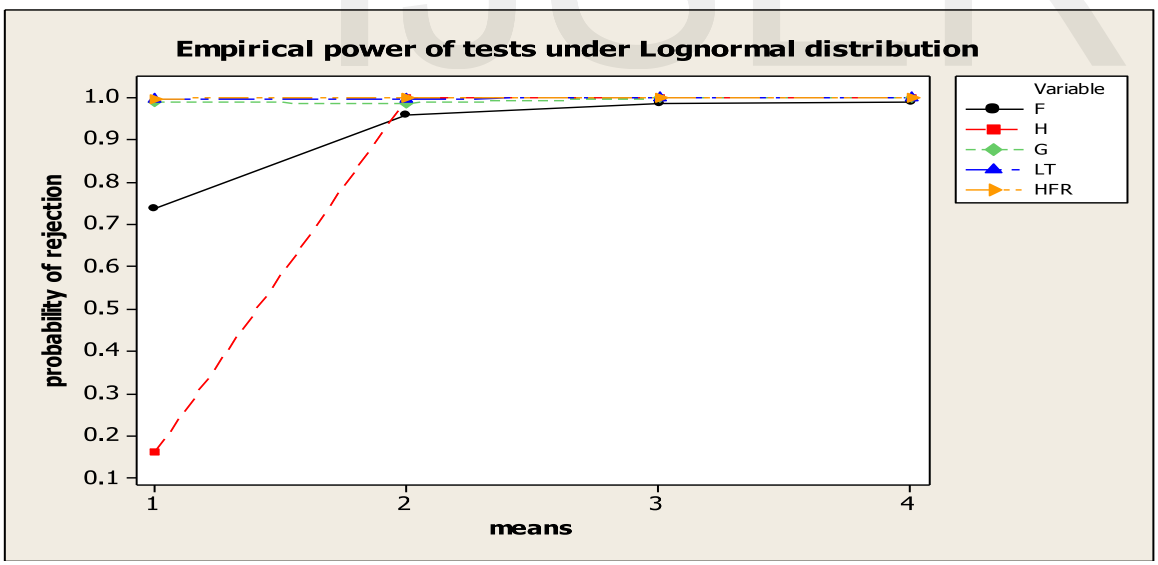

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Table 1.6 Empirical Level and power of tests under Exponential distribution:

Sample sizes n_i	Location parameter μ_i	F 5%	F 1%	H 5%	H 1%	G 5%	G 1%	LT 5%	LT 1%	HFR 5%	HFR 1%
10 10 10	0 0 0	.0353	.0057	.0454	.0068	.0417	.0043	.0482	.0067	.0451	.0059
	0 -.2 .2	.1176	.0300	.1814	.052	.1554	.0268	.1174	.0262	.1898	.0482
	0 -.5 .5	.2759	.0902	.6742	.3824	.4094	.1543	.4501	.2046	.6970	.3646
	0 -1 1	.7665	.4559	.9834	.9144	.7305	.4156	.8928	.6954	.9830	.8633
	0 -1.5 1.5	.9212	.7287	.9996	.9954	.8821	.6101	.9879	.9277	.9995	.9793
10 15 20	0 0 0	.0389	.0050	.0456	.0066	.0430	.0070	.0467	.0075	.0451	.0069
	0 -.2 .2	.1798	.0648	.3186	.1244	.2610	.0846	.1602	.0506	.3450	.1348
	0 -.5 .5	.4228	.1469	.8922	.7224	.6941	.4009	.6523	.4148	.9211	.7703
	0 -1 1	.9095	.6806	.9998	.9970	.9307	.7611	.9777	.9238	.9993	.9947
	0 -1.5 1.5	.9656	.8568	1.000	1.000	.9831	.9152	.9988	.9932	1.000	.9998
10 10 10 10	0 0 0 0	.0338	.0057	.0444	.0050	.0414	.0053	.0453	.0069	.0421	.0050
	-.1 .1 -.2 .2	.1092	.0266	.1872	.0496	.2300	.0540	.1442	.0370	.2870	.0886
	-.5 .5 -1 1	.7841	.4927	.9950	.9634	.9388	.7428	.9940	.9702	.9998	.9962
	-1 1 -1.5 1.5	.9302	.7623	.1.000	1.000	.9936	.9228	1.000	.9998	1.000	1.000
	-1.5 1.5 -2 2	.9486	.8157	1.000	1.000	.9994	.9962	1.000	1.000	1.000	1.000
10 15 20 25	0 0 0 0	.0420	.0082	.0484	.0078	.0492	.0063	.0470	.0082	.0463	.0083
	-.1 .1 -.2 .2	.1970	.0678	.3718	.1646	.5226	.2620	.2638	.1036	.5910	.3302
	-.5 .5 -1 1	.9595	.8302	1.000	1.000	.9994	.9952	1.000	.9996	1.000	1.000
	-1 1 -1.5 1.5	.9848	.9280	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	-1.5 1.5 -2 2	.9883	.9435	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Fig. 1.9 Empirical power of tests under Exponential distribution for n_1=n_2=n_3=10 at 5% level:

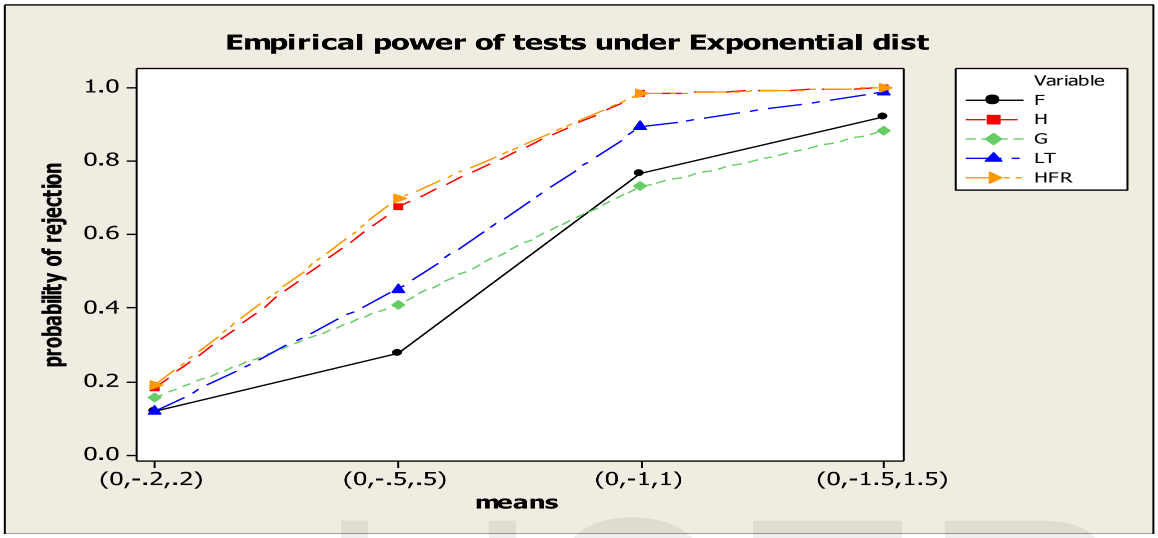


Fig. 1.10 Empirical power of tests under Exponential test for n_1=10, n_2=15, n_3=20, n_4=25 at 5% level:

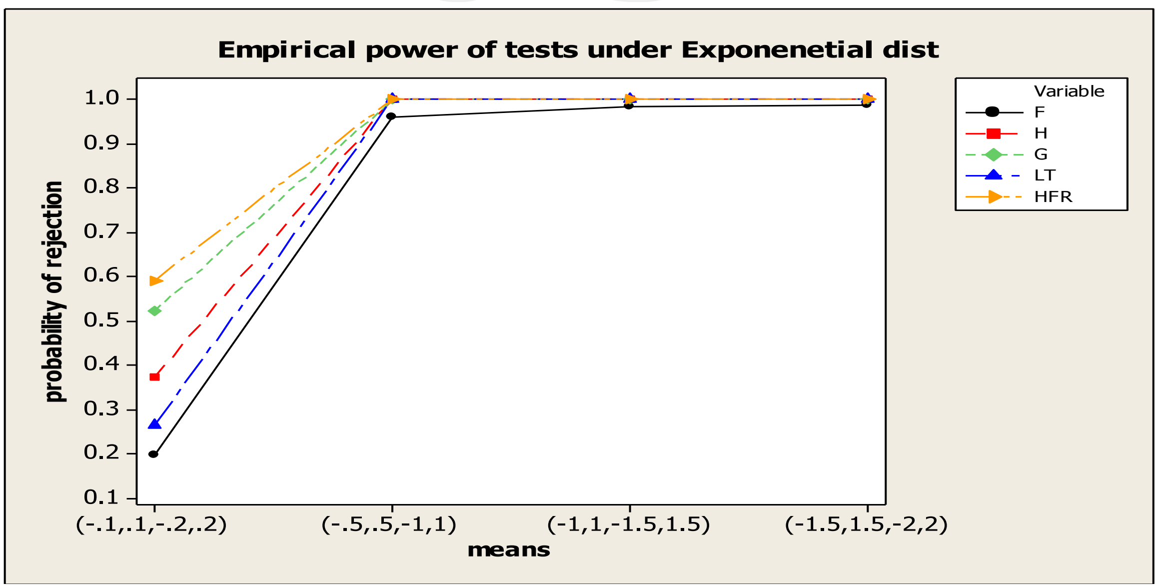


Table 1.7 Empirical Level and power of tests under Double Exponential Distribution:

Sample sizes n_i	Location parameter μ_i	F 5%	F 1%	H 5%	H 1%	G 5%	G 1%	LT 5%	LT 1%	HFR 5%	HFR 1%
10 10 10	0 0 0	.0470	.0086	. 0454	.0068	.0362	.0048	.0455	.0076	.0430	.0076
	0 -.2 .2	.0778	.0171	.0870	.0176	.0422	.0074	.0742	.0110	.0788	.0148
	0 -.5 .5	.2718	.1001	.3186	.1138	.0845	.0167	.3382	.1278	.2633	.0783
	0 -1 1	.7723	.5436	.8236	.5632	.2725	.0996	.8388	.5985	.7173	.4386
	0 -1.5 1.5	.9758	.9086	.9828	.9162	.5373	.2784	.9840	.9196	.9415	.7973
10 15 20	0 0 0	.0498	.0077	.0456	.0066	.0443	.0073	.0462	.0072	.0477	.0060
	0 -.2 .2	.1079	.0288	.1278	.0362	.0644	.0108	.0916	.0194	.1126	.0324
	0 -.5 .5	.4373	.2182	.5406	.2774	1677	.0506	.5622	.3020	.4660	.2264
	0 -1 1	.9476	.8429	.9772	.8942	.5435	.2757	.9800	.9149	.9443	.8228
	0 -1.5 1.5	.9998	.9950	.9998	.9974	.8594	.6414	.9999	.9975	.9987	.9900
10 10 10 10	0 0 0 0	.0462	.0070	.0444	.0050	.0363	.0053	.0454	.0068	.0436	.0070
	-.1 .1 -.2 .2	.0775	.0156	.0853	.0144	.0428	.0074	.0894	.0168	.0710	.0132
	-.5 .5 -1 1	.8166	.6050	8830	.6504	.2663	.0837	.8953	.6941	.7695	.4910
	-1 1 -1.5 1.5	.9942	.9683	.9984	.9796	.5533	.1906	.9978	.9844	.9800	.9068
	-1.5 1.5 -2 2	.9999	.9989	1.000	.9996	.7888	.2780	1.000	.9999	.9986	.9884
10 15 20 25	0 0 0 0	.0475	.0095	.0484	.0078	.0414	.0075	.0447	.0080	.0434	.0071
	-.1 .1 -.2 .2	.1106	.0293	.1392	.0378	.0648	.0114	.1506	.0460	.1254	.0338
	-.5 .5 -1 1	.9842	.9430	.9970	.9742	.6666	.4133	.9979	.9829	.9873	.9462
	-1 1 -1.5 1.5	1.000	.9999	1.000	1.000	.9558	.8449	1.000	1.000	.9999	.9996
	-1.5 1.5 -2 2	1.000	1.000	1.000	1.000	.9975	.9766	1.000	1.000	1.000	1.000

Fig.1.11 Empirical power of tests under Double Exponential distribution for $n_1=n_2=n_3=10$ at 5% level:

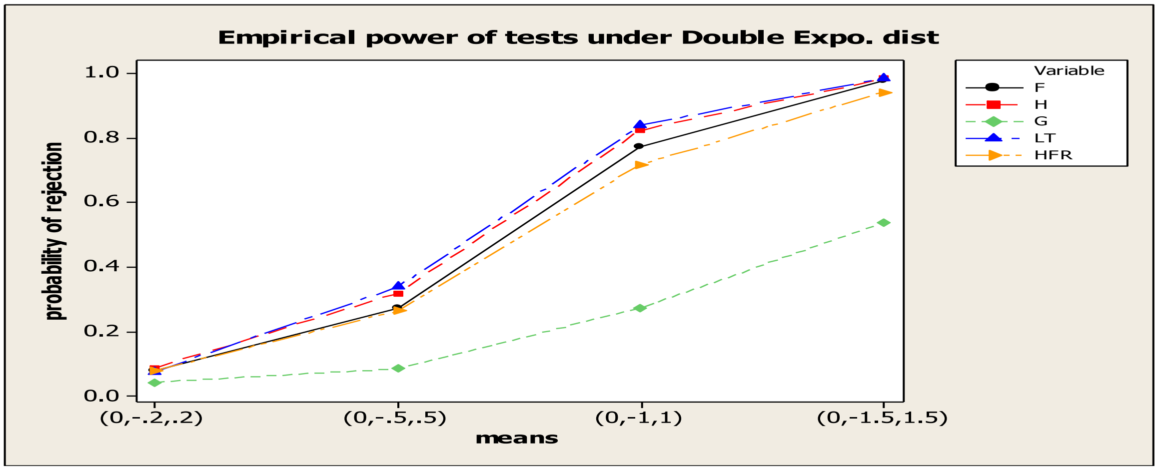
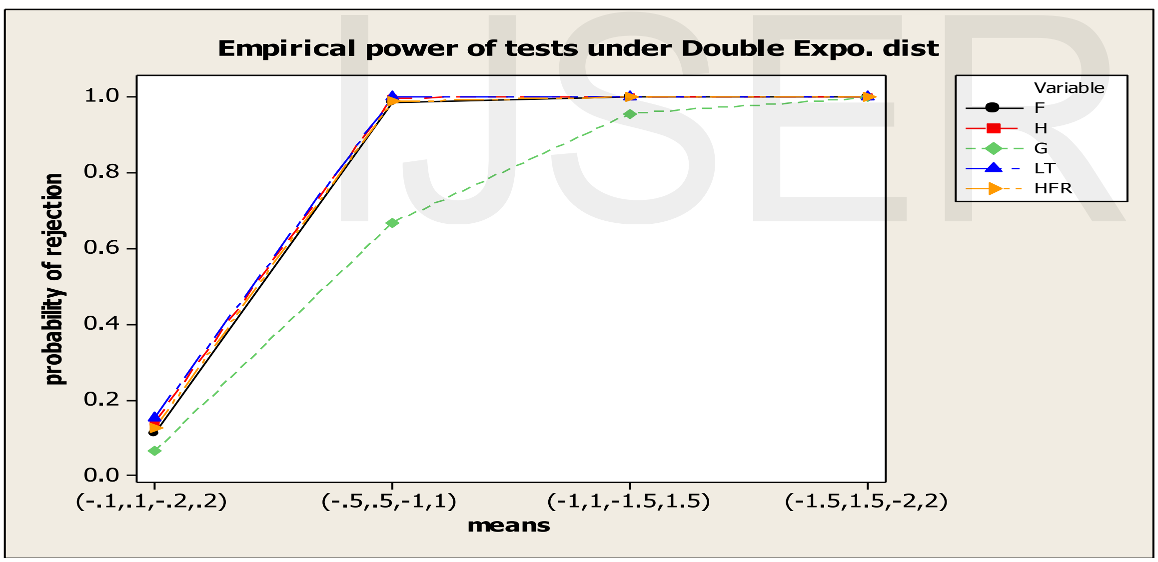


Fig.1.12 Empirical power of tests under Double Exponential distribution for $n_1=10$, $n_2=15$, $n_3=20$, $n_4=25$ at 5% level:



1.5 Discussion:

From Table 1.2 it is seen that empirical level of almost all tests satisfies the nominal level. In case of power , F- test seems to be more powerful than other tests followed by Kruskal –Wallis test. But as the location shift , sample size increases power of all the tests going to be almost equal.

In case Cauchy distribution, only Kruskal-Wallis and Long-tail test satisfies the nominal levels under the null situations. F-test not at all satisfies the nominal level. However,G, HFR slightly better than F-test. Power of Long –tail test (LT) seems to be the highest of all the tests discussed here.

Table 1.4 shows the empirical level and power of six tests under logistic distribution. Here we have observe that all the test satisfies the nominal levels. It is seen that power of F-test and Kruskal –Wallis tests are almost similar. However, KW tests are slightly higher in some situations. Out of three score tests, power LT test is higher than other two tests.

Table 1.5 shows the empirical level and power of tests under lognormal distribution. It is seen that except F –test all other tests satisfy the nominal level approximately. Here we have seen that power of HFR test is more than other tests. Power of F and G test are found to be less than other tests.

Table 1.6 shows empirical levels and powers of tests under exponential distribution . Here we have found similar results as lognormal distribution. Since both are right-skewed distribution that why we get similar results.

From 1.7 , we have found the empirical levels and power of the six tests. It is observe that except G test, empirical level of other test are closed to nominal levels. It is also clear that power of LT test is the highest of all , followed by KW and F test and HFR respectively.

1.6 Conclusions:

From the above results we can conclude that ,F test is suitable for the normal distribution. For log tailed distribution LT test and H test is more preferable than other test. G test is suitable for short tailed distribution and HFR test is preferable for the right-skewed distribution. From these results it is clear that prior information regarding the observation distribution help in choosing the appropriate test. So, adaptive procedure certainly help the practioner for appropriate test selection and help to arrive at right conclusion.

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